As an induction step, assume that $f^2(a_n) \geq 2nf(1)$ for some $n \geq 1$. Then

$$\begin{aligned} f^2(a_{n+1}) &= f^2\left(a_n + \frac{1}{a_n}\right) \\ &\geq f^2(a_n) + 2f(1) + f^2\left(\frac{1}{a_n}\right) \\ &\geq f^2(a_n) + 2f(1) \;\geq\; 2(n+1)f(1)\,, \end{aligned}$$

completing the induction. Hence $f^2(a_n) \geq 2nf(1)$ for all $n \geq 1$, contradicting the facts that $f(1) > 0$ and f is bounded.

And to complete our files for the *Corner*, we look at a problem of the Taiwan Mathematical Olympiad, Selected Problems 2005, given in [2008: 21–22].

1. A $\triangle ABC$ is given with side lengths a, b, and c. A point P lies inside $\triangle ABC$, and the distances from P to the three sides are p, q, and r, respectively. Prove that

$$R \;\leq\; \frac{a^2+b^2+c^2}{18\sqrt[3]{pqr}}\,,$$

where R is the circumradius of $\triangle ABC$. When does equality hold?

Solved by Arkady Alt, San Jose, CA, USA; Michel Bataille, Rouen, France; and George Tsapakidis, Agrinio, Greece. We give Bataille's write-up.

Let F denote the area of $\triangle ABC$. We have the well-known relation $2F = \frac{abc}{2R}$, but also from the definition of p, q, and r we have the equation $2F = ap + bq + cr$. Thus, the proposed inequality is equivalent to

$$\frac{abc}{2(ap+bq+cr)} \;\leq\; \frac{a^2+b^2+c^2}{18\sqrt[3]{pqr}}$$

or

$$(a^2+b^2+c^2)(ap+bq+cr) \;\geq\; 9abc\sqrt[3]{pqr}\,. \qquad (1)$$

By the AM–GM Inequality,

$$a^2+b^2+c^2 \;\geq\; 3\sqrt[3]{a^2b^2c^2} \quad \text{and} \quad ap+bq+cr \;\geq\; 3\sqrt[3]{abcpqr}\,,$$

and the inequality (1) now follows from

$$(a^2+b^2+c^2)(ap+bq+cr) \;\geq\; 9\sqrt[3]{a^2b^2c^2}\cdot\sqrt[3]{abc}\cdot\sqrt[3]{pqr}\,.$$

That completes the *Corner* for this number, and this Volume. As Joanne Canape, who has been translating my scribbles into beautiful LaTeX has decided that twenty-plus years is enough, I want to thank her too for all the help over the time we've worked together.